Objective type questions (**with answers**) for
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1. Let $d : X \times X \to \mathbb{R}$, where X be a non-empty set. d is a metric if

   (a) $d(x, y) \geq 0$ for all $x, y \in X$ and $d(x, y) \leq d(x, z) + d(y, z)$, $\forall x, y, z \in X$.
   (b) $d(x, y) = 0$ iff $x = y$ and $d(x, y) \leq d(x, z) + d(y, z)$, $\forall x, y, z \in X$.
   (c) $d(x, y) = d(y, x)$, $\forall x, y \in X$ and $d(x, y) \leq d(x, z) + d(y, z)$, $\forall x, y, z \in X$.
   (d) none of these.

   Answer (b)

2. Let $\mathbb{R}$ be equipped with usual metric. Then the set $\mathbb{I}$ of all irrational numbers are dense in $\mathbb{R}$ since

   (a) $\mathbb{I}$ is uncountable
   (b) $\mathbb{I}$ is the complement of a countable set
   (c) $\mathbb{I}$ is the complement of a dense set
   (d) $\mathbb{I}$ is an infinite set

   Answer (b)

3. Let (X, d) be a metric space and $A \subseteq X$. Then

   (a) A is connected $\Rightarrow$ Int A is connected
   (b) Int A is connected $\Rightarrow$ A is connected
   (c) A is connected $\Rightarrow$ $\overline{A}$ is connected
   (d) $\overline{A}$ is connected $\Rightarrow$ A is connected

   Answer (c)

4. Let f be a twice differentiable function on $\mathbb{R}$ such that $f''(x) > 0$, $\forall x \in \mathbb{R}$. Then $f(x) = 0$ has

   (a) exactly two solution
   (b) a positive solution if $f(0) = 0$ and $f'(0) = 0$
   (c) no positive solution if $f(0) = 0$ and $f'(0) > 0$
   (d) no positive solution if $f(0) = 0$ and $f'(0) < 0$

   Answer (c)

5. Let f be a continuously differentiable function on $[a, b]$ such that $|f''(x)| \leq K$, $\forall x \in [a, b]$. For a partition $P = \{a = a_0 < a_1 < \cdots < a_n = b\}$, if $U(P, f)$ and $L(P, f)$ denotes the upper and lower Riemann sums of f over P, then

   (a) $|L(P, f)| \leq K(b - a) \leq |U(P, f)|$
   (b) $U(P, f) - L(P, f) \leq K(b - a)$
   (c) $U(P, f) - L(P, f) \leq K\|P\|$, where $\|P\| = \max(a_i - a_{i-1})$
   (d) $U(P, f) - L(P, f) \leq K\|P\|(b - a)$

   Answer (c)

6. If f is a monotonically increasing function on $\mathbb{R}$, then

   (a) $\lim\limits_{x \to a} f(x)$ exists $\forall a$
   (b) $\lim\limits_{x \to a^+} f(x) \geq \lim\limits_{x \to b^-} f(x)$, for $a < b$
   (c) f can have discontinuity of first kind
   (d) f can have discontinuity of second kind

   Answer (d)

7. Let X be a connected subset of $\mathbb{R}$ such that every element of X is an irrational number, then the cardinality of X is
   (a) infinite
   (b) countably infinite
   (c) 2
   (d) 1

   Answer (d)

8. If (a_n) and (b_n) sequences of positive real numbers such that arithmetic average and geometric average of a_n and b_n converge to a same limit, then both (a_n), (b_n)

(a) are convergent but the limits may not be equal
(b) converge to a same limit
(c) need not be convergent
(d) are not convergent

Answer (b)

9. The value of $\sum_{1}^{\infty} \frac{1}{n^2}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi^2}{3}$ (d) $\frac{\pi^2}{6}$

Answer (d)

10. Let $X = \{1, 2, 3, 4, 5\}$ and $\mathcal{T} = \{\varnothing, X, \{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$ Then which of the following is the limit point of $\{3, 4, 5\}$

(a) 1 (b) 2 (c) 3 (d) 4

Answer (d)

11. If $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ are topological spaces, then $\{U \times V : U \in \mathcal{T}_X, \mathcal{T}_X \in \mathcal{T}_Y\}$ is a

(a) a topology (b) basis (c) subbasis but not a basis (d) not a subbasis

Answer (b)

12. If $f : (\mathbb{R}, \mathcal{T}_1) \to (\mathbb{R}, \mathcal{T}_2)$ is defined by $f(x) = x,\ \forall x \in \mathbb{R}$(the set of all real numbers), where $\mathcal{T}_1$ is the usual topology and $\mathcal{T}_2$ is the discrete topology then

(a) f is both continuous and open
(b) f is open but not continuous
(c) f is continuous but not open
(d) f is neither open not continuous

Answer (b)

13. Let $(X, \mathcal{T})$ be a topological space where, X is a finite set. Then which of the following statements is not true? $\mathcal{T}$ is discrete if and only if X is a

(a) T_0 - space (b) T_1 - space (c) T_2 - space (d) T_3 - space

Answer (a)

14. Let $(X_\alpha, \tau_\alpha)_{\alpha \in I}$ be an arbitrary collection of topological spaces. Then which of the following statements is not true?

(a) $\prod_{\alpha \in I} X_\alpha$ is connected if each X_α is connected
(b) $\prod_{\alpha \in I} X_\alpha$ is compact if each X_α is compact
(c) each X_α is compact if $\prod_{\alpha \in I} X_\alpha$ is compact
(d) $\prod_{\alpha \in I} X_\alpha$ is normal if each of X_α is normal.

Answer (d)

15. Consider $\mathbb{R}^k$ with the topology induced by the Euclidean norm. Which of the following statements is not true?

(a) $\mathbb{R}^k$ is second countable
(b) $\mathbb{R}^k$ is first countable
(c) $\mathbb{R}^k$ is of first category
(d) $\mathbb{R}^k$ is of second category

Answer (c)

16. Let $f : (X_1, \mathcal{T}_1) \to (X_2, \mathcal{T}_2)$ be a continuous, one-to-one and onto map. f is a homeomorphism if

(a) X_1 is Hausdorff and X_2 is compact
(b) X_1 is Hausdorff or X_2 is compact
(c) X_1 is compact and X_2 is Hausdorff
(d) X_1 is compact or X_2 is Hausdorff

Answer (c)

17. If $(\mathbb{R}^*, \star)$ is a group with respect to $a \star b = \dfrac{ab}{2}$ then the inverse of a is

(a) $\dfrac{1}{a}$ (b) $\dfrac{2}{a}$ (c) $\dfrac{3}{a}$ (d) $\dfrac{4}{a}$

Answer (b)

18. The number of elements in the cyclic subgroup $\langle 2 \rangle$ in $(\mathbb{Z}_{18}, \oplus)$ is

(a) 3 (b) 6 (c) 9 (d) 18

Answer (d)

19. The group of order 25 is

(a) cyclic (b) abelian (c) abelian but not cyclic (d) neither abelian nor cyclic

Answer (c)

20. If R is a ring such that $a^2 = a$ for all $a \in R$ then which of the following is not true?

(a) $a + b = 0 \Rightarrow a = -b$ (b) $a + b = 0 \Rightarrow a = b$ (c) $a + b = 0 \Rightarrow ab = ba$ (d) $a + b = 0 \Rightarrow a = b = 0$

Answer (a)

21. The characteristic of the ring $(\mathscr{P}(S), \Delta, \cup)$ is

(a) 0 (b) 2 (c) 3 (d) 5

Answer (b)

22. The prime ideal of $\mathbb{Z}$ is

(a) $\langle 1 \rangle$ (b) $\langle 2 \rangle$ (c) $\langle 4 \rangle$ (d) $\langle 6 \rangle$

Answer (b)

23. Let K be the field of complex numbers and F the field of real numbers. Then $O[G(K, F)]$ is

(a) infinite (b) 1 (c) 2 (d) none of these

Answer (d)

24. Which of the following rings is a PID?

(a) $\mathbb{Q}[X, Y]/(X)$ (b) $\mathbb{Z} + \mathbb{Z}$ (c) $\mathbb{Z}[X]$ (d) $M_2(\mathbb{Z})$

Answer (a)

25. The polynomial $x^3 - 312312x + 123123$ is irreducible in $F[X]$ if F is

(a) $\mathbb{F}_3$ (b) $\mathbb{F}_{13}$ (c) $\mathbb{F}_{17}$ (d) $\mathbb{Q}$

Answer (d)

26. The normed space $(C_{00}, \|\cdot\|_\infty)$ is

(a) a Banach space (b) not a Banach space
(c) a Hilbert space (d) Banach space not a Hilbert space

Answer (b)

27. If V is the inner product space of all polynomials of the form $p : [0, 1] \to \mathbb{R}$ such that $p(x) = ax + b$, $a, b \in \mathbb{R}$, with the inner product $\langle p, q \rangle = \int_0^1 p(x)q(x)\,dx$, then an orthonormal basis of V is

(a) $\{1, x\}$ (b) $\{1, x\sqrt{3}\}$ (c) $\{1, (2x-1)\sqrt{3}\}$ (d) $\{1, x - \frac{1}{2}\}$.

Answer (c)

28. If $f : \mathbb{R}^n \to \mathbb{R}$ is a linear map the set $\{f(x_1, x_2, \ldots, x_n) : x^2 + x^2 + \cdots + x_n^2 \leq 1\}$ equals

(a) $[-a, a]$ for some $a \geq 0$ (b) $[0, 1]$ (c) $[0, a]$ for some $a \geq 0$ (d) $[a, b]$ for some $a < b$

Answer (a)

29. Let $T : X \to Y$ be a continuous linear transformation between normed spaces. Then T is continuous if

(a) X is of finite dimension
(b) Y is of finite dimension
(c) X is of infinite dimension
(d) Y is of infinite dimension

Answer (a)

30. Let C_{00} be the space of all real sequences which are eventually zero. If $\|(x_n)\|_p = \left\{ \sum_{n=1}^{\infty} |x_n|^p \right\}^{1/p}$ for $1 \le p < \infty$ and $\|(x_n)\|_\infty = \sup\{|x_n| : n \in \mathbb{N}\}$. Then the identity transformation $I : (C_{00}, \|\cdot\|_a) \to (X, \|\cdot\|_b)$ is

(a) continuous when $a = \infty$ and $b = 1$
(b) I is not continuous when $a = \infty$ and $b = 1$
(c) I is continuous when $a = 2$ and $b = 1$
(d) I is not continuous when $a = 3$ and $b = 4$

Answer (b)

31. The dimension of the subspace spanned by $(1, 0, 2, 0), (2, 0, 1, 0), (1, 0, 1, 0)$ in $\mathbb{R}^4$ is
(a) 1 (b) 2 (c) 3 (d) 4

Answer (c)

32. The matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T(a, b, c) = (3a+b+2c, a+5b, 2a+2c)$ with respect to the standard basis is
(a) $\begin{pmatrix} 3 & 1 & 2 \\ 1 & 5 & 0 \\ 2 & 0 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 1 & 2 \\ 1 & 0 & 5 \\ 2 & 5 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & 1 & 2 \\ 1 & 5 & 0 \\ 2 & 0 & 5 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 5 & 0 \\ 2 & 0 & 2 \end{pmatrix}$

Answer (c)

33. The dimension of the subspace spanned by $(1, 0, 2, 0), (2, 0, 1, 0), (1, 0, 1, 0)$ in $\mathbb{R}^4$ is
(a) 1 (b) 2 (c) 3 (d) 4

Answer (c)

34. If $M_n(\mathbb{R})$ is the space of all $n \times n$ real matrices, and $T : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ is a liner transformation such that $T(A) = 0$, for all symmetric or skew symmetric matrices A, then the rank of A is
(a) 0 (b) n (c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$

Answer (a)

35. If H_n is the real vector space of all matrices of order $n \times n$ satisfying $a_{i,j} = a_{r,s}$ if $i + j = r + s$, then dimension of H_n is
(a) n^2 (b) $n^2 - n + 1$ (c) $2n + 1$ (d) $2n - 1$

Answer (d)

36. The radius of convergence of the power series $\sum_{n=1}^{\infty} z^{n!}$ is
(a) 0 (b) ∞ (c) 1 (d) a real number greater than 1

Answer (c)

37. If x and y are complex numbers such that $|x + y| = |x| + |y|$. Then
(a) x and y are positive real numbers
(b) x and y have same imaginary parts
(c) either $x = \alpha y$ or $y = \alpha x$ for some real $\alpha \ge 0$
(d) either $x = \alpha y$ or $y = \alpha x$ for some real number α

Answer (c)

38. The radius of convergence of $\sum_{n=0}^{\infty} a_n z^n$, where $a_n = m,\ 0 \le m \le 4$ and $n \equiv m \pmod 5$, is

(a) 0 (b) ∞ (c) 4 (d) $\frac{1}{4}$

Answer (d)

39. If $f(z) = \dfrac{z^4}{(z-1)(z-2)(z-3)}$, the residue of f at $z = 1$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) 0 (d) 2

Answer (a)

40. The value of $\int\limits_{|z|=4} \dfrac{z^2}{z-2} dz$

(a) $\frac{2}{\pi i}$ (b) $\frac{2}{\pi}$ (c) 8π (d) $8\pi i$

Answer (b)

41. For a binomial distribution with parameters $(n,\ p)$ has mean $= 27$ and variance $= 18$. Find n.
(a) 18 (b) 27 (c) 81 (d) 162

Answer(d)

42. The probability that at least one of A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3 then $p(A') + p(B')$ (where B' means complement of B) is
(a) 0.9 (b) 1.15 (c) 1.1 (d) 1.2

Answer (c)

43. If the mean and variance of a binomial variate X are 2 and 1 respectively then the probability that X takes a value at least one is
(a) 2/3 (b) 4/5 (c) 7/8 (d) 15/16

Answer(d)

44. Let $X_1, X_2, \ldots$ be i.i.d standard normal random variables and let $T_n = \frac{X_1^2 + X_2^2 + \cdots + X_n^2}{n}$. Then the limiting distribution of

(a) $T_n - 1$ is χ^2 with 1 degree of freedom (b) $\frac{T_n - 1}{\sqrt{n}}$ is normal with mean 0 variance 2
(c) $\sqrt{n}(T_n - 1)$ is χ^2 with 1 degree of freedom (d) $\sqrt{n}(T_n - 1)$ is normal with mean 0 variance 2

Answer (d)

45. Men and a women independently arrive in a queue according to Poisson process with rate λ_1 and λ_2, respectively. The probability that the first arrival in the queue is a man is
(a) $\frac{\lambda_1}{\lambda_1+\lambda_2}$ (b) $\frac{\lambda_2}{\lambda_1+\lambda_2}$ (c) $\frac{\lambda_1}{\lambda_2}$ (d) $\frac{\lambda_2}{\lambda_1}$
Answer (a)

46. If $y_1(x)$ and $y_2(x)$ be two solutions of $\frac{dy}{dx} = y + 17$ with initial conditions $y_1(0) = 0,\ y_2(0) = 1$. Then

(a) y_1 and y_2 will never intersect (b) y_1 and y_2 will intersect at 17
(c) y_1 and y_2 will intersect at e (d) y_1 and y_2 will intersect at 1

Answer (a)

47. The partial differential equation $y\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} = 0$ is hyperbolic in

(a) the second and fourth quadrants
(b) the first and fourth quadrants
(c) the second and third quadrants
(d) the first and fourth quadrants

Answer (a)

48. The particular solution of $\log(dy/dx) = 3x + 4y, y(0) = 0$ is
(a) $e^{3x} + 3e^{-4y} = 4$ (b) $4e^{3x} - 3e^{-4y} = 3$ (c) $3e^{3x} + 4e^{4y} = 7$ (d) $4e^{3x} + 3e^{-4y} = 7$

Answer (d)

49. The complete solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}$ is
(a) $Ae^{x} + Be^{2x} + xe^{2x}$ (b) $Ae^{-x} + Be^{-2x} + xe^{2x}$ (c) $Ae^{x} + Be^{2x} + xe^{x}$ (d) $Ae^{-x} + Be^{-2x} + e^{2x}$

Answer (a)

50. The PDE $\frac{\partial^2 u}{\partial y^2} - y\frac{\partial^2 u}{\partial x^2} = 0$ has
(a) two families of real characteristic curves for $y < 0$
(b) no real characteristic for $y < 0$
(c) vertical lines as a family of real characteristic curves for $y = 0$
(d) branches of quadratic curves as characteristics for $y \neq 0$

Answer (c)